High-energy effective action from scattering of QCD shock waves

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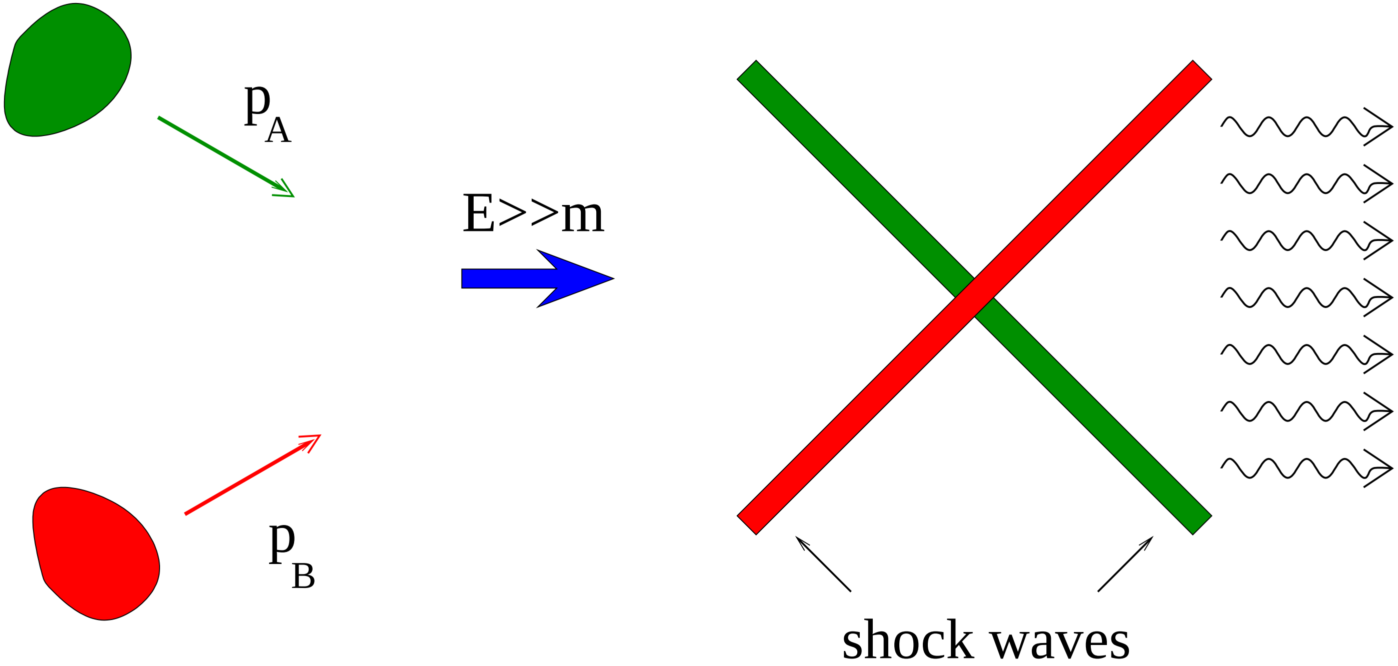
Sinaia, 29 June 05

High-energy scattering as a collision of shock waves

A typical hadron-hadron collision viewed from the c.m. frame has the form of scattering of two shock waves.

Regge limit: $E \gg$ everything else



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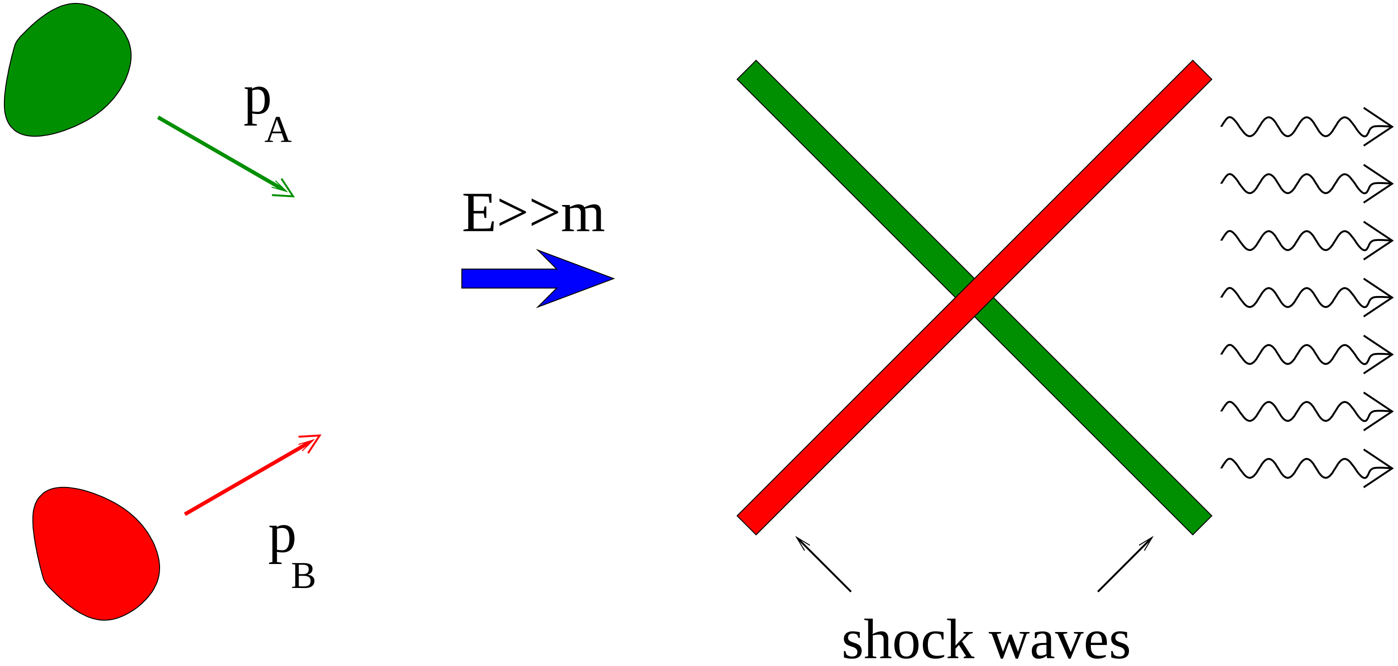

- **Big Q: Produced particles/fields $\Leftrightarrow$ S_{eff}?**

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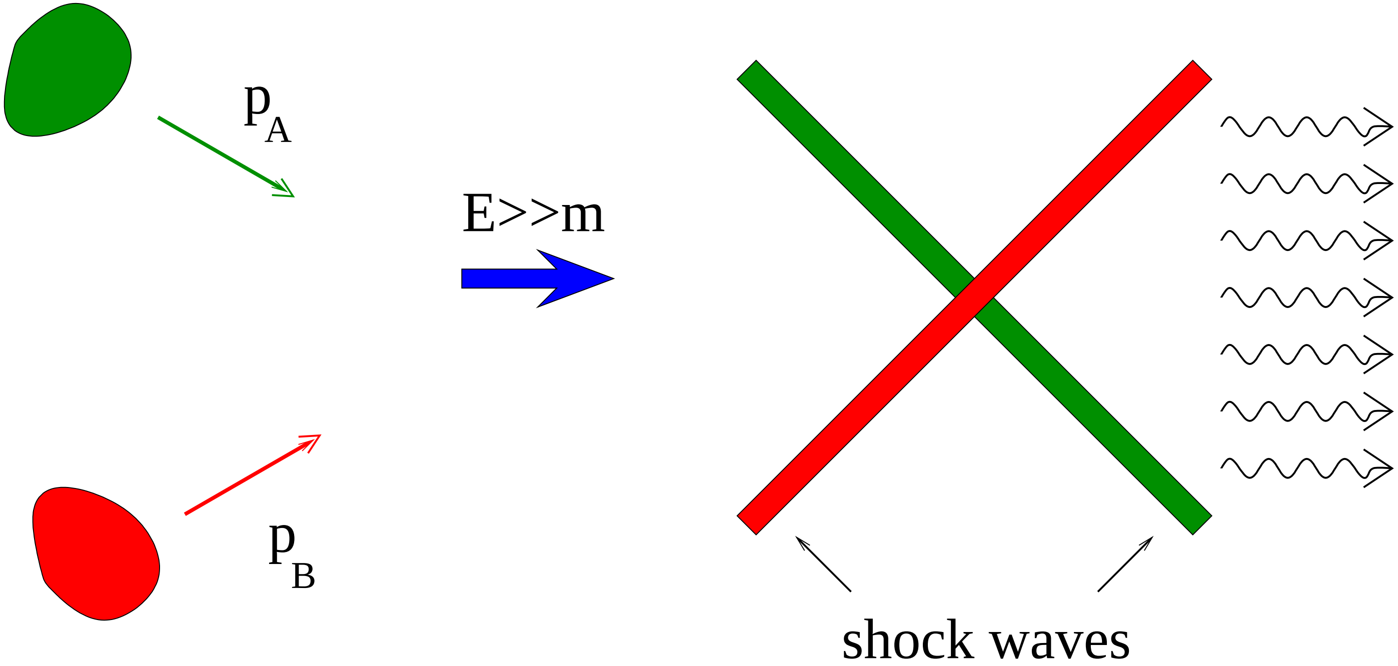
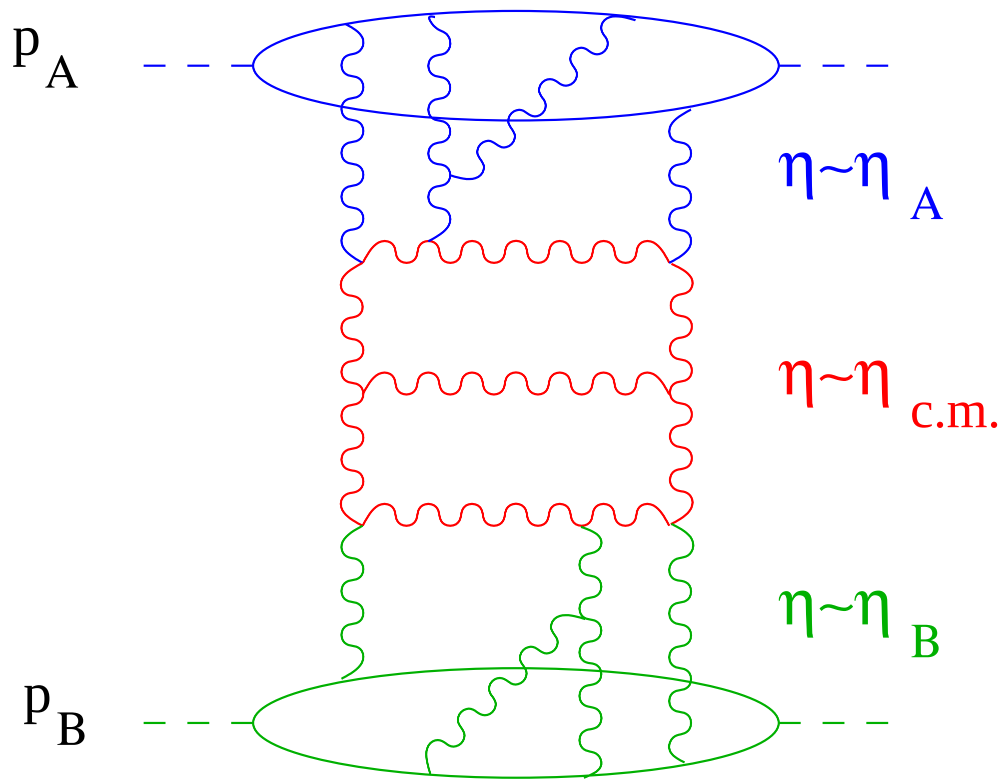


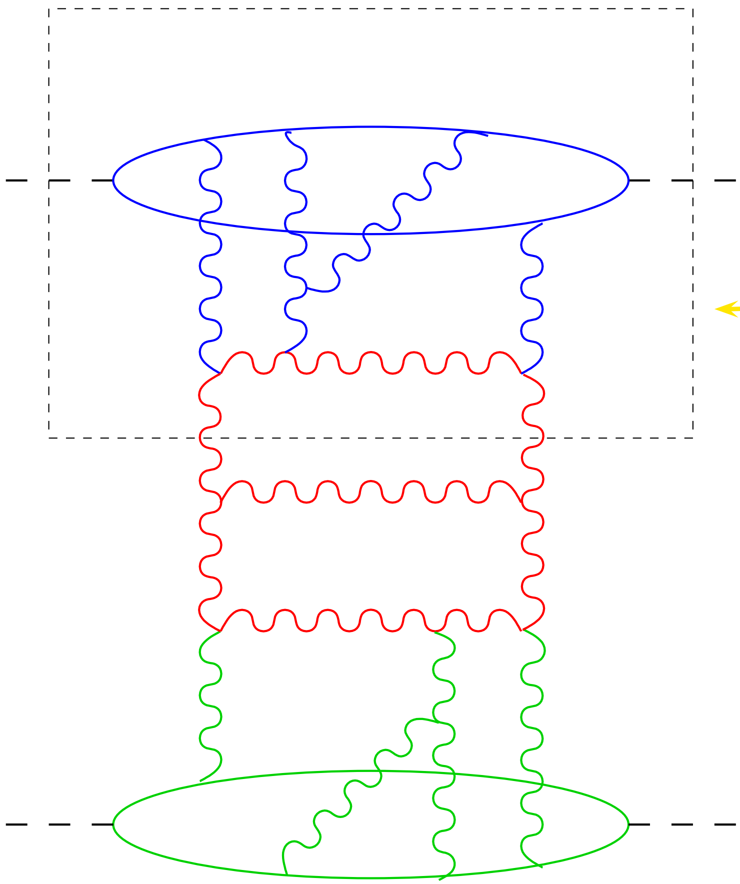
- **Q # 0: What is a scattering of two QCD shock waves?**
- **Big Q: Produced particles/fields $\Leftrightarrow$ $S_{\rm eff}$?**

Rapidity factorization



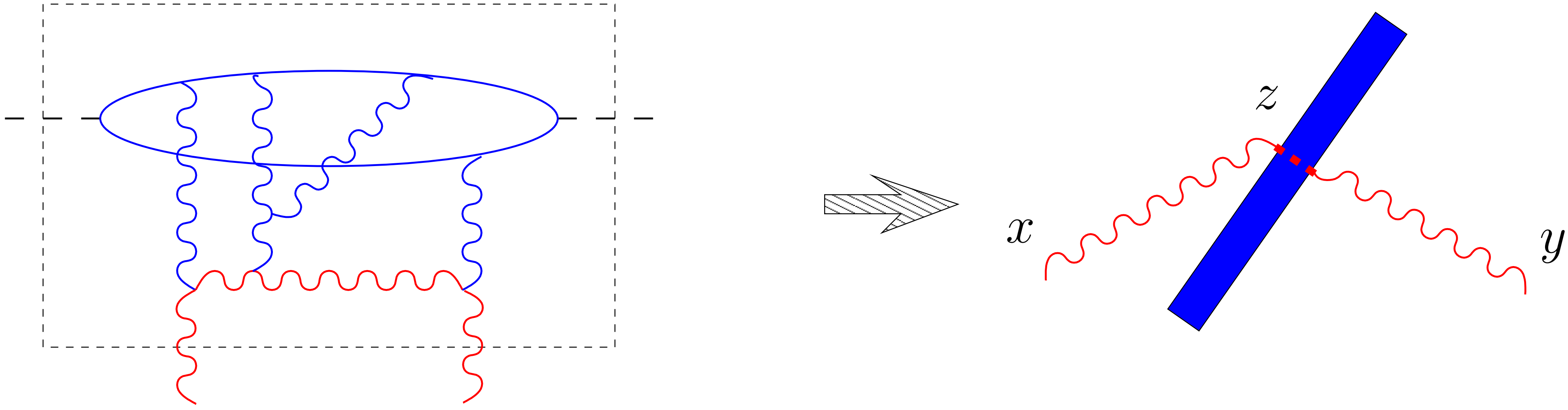
At first, we integrate over "red" gluons moving with rapidities in the central region $\eta \sim \eta_{\rm c.m.}$.
They interact with the "external" fileds (to be integrated over later) with rapidities $\eta \sim \eta_A$ and $\eta \sim \eta_B$

Rapidity factorization



Consider the propagation of the red gluon in the background of blue gluons with greater rapidity

Fast-moving hadron $\Rightarrow$ QCD shock wave



Fast-moving (blue) fileds shrink into a pancake $A_+ \sim \delta(x_-)$.

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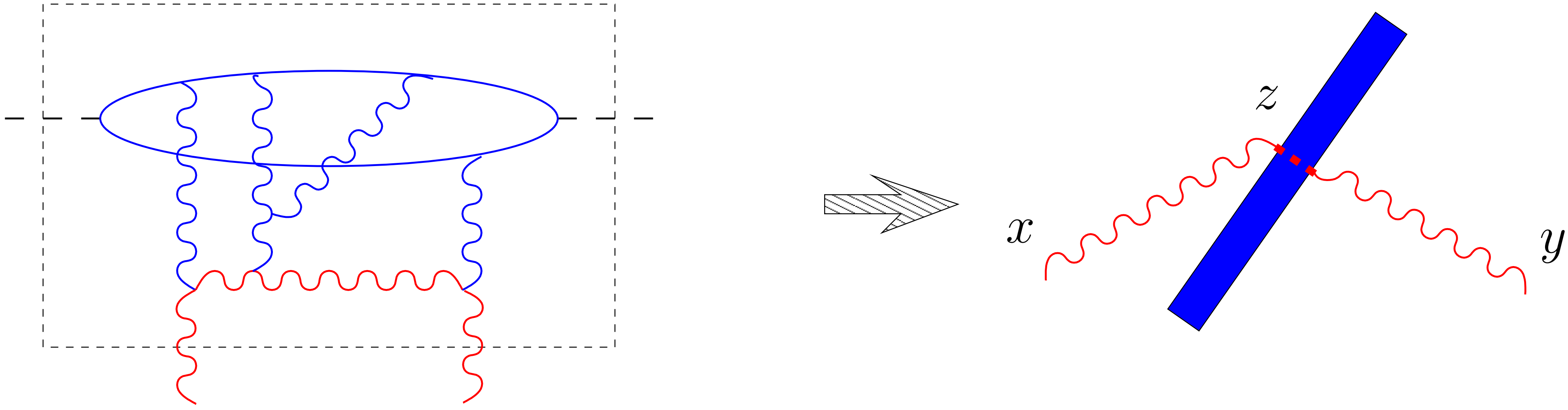

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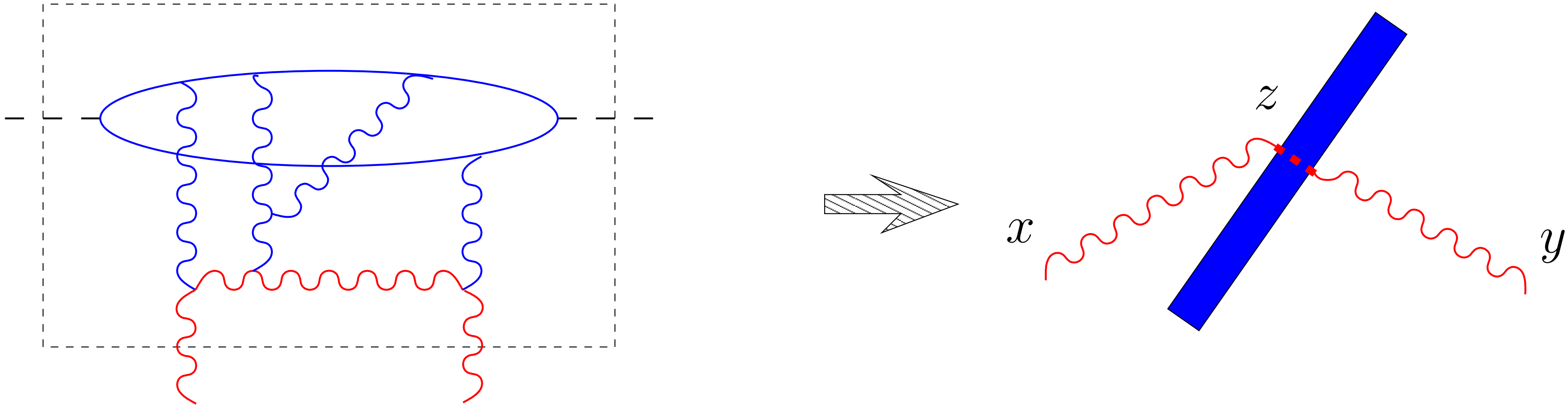
Interaction with the shock wave is instantaneous
$\Rightarrow$ no time to deviate in transverse plane

Fast-moving hadron ⇒ QCD shock wave



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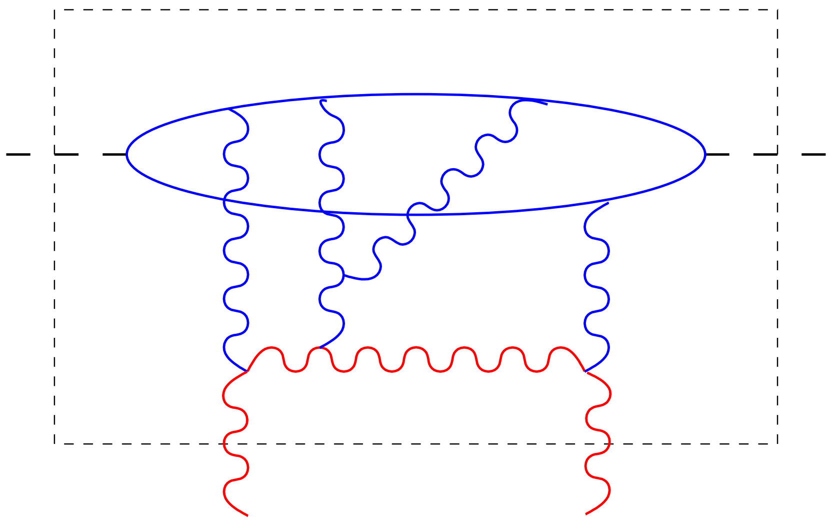
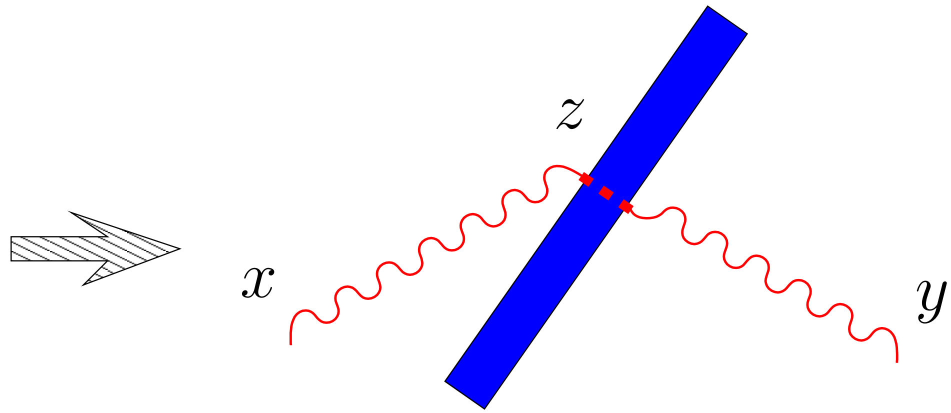
⇒ no time to deviate in transverse plane
⇒ the interaction is described by the *Wilson line*

$$V_z = [\infty p_2 + z_\perp, -\infty p_2 + z_\perp], \qquad [x, y] \equiv Pe^{ig\int_0^1 du (x-y)^\mu A_\mu(ux+(1-u)y)}$$

Fast-moving hadron ⇒ QCD shock wave





$$G(x,y) = \int dz \frac{1}{(x-z)^2} V(z_\perp) \delta(z_-) \frac{1}{(z-y)}$$

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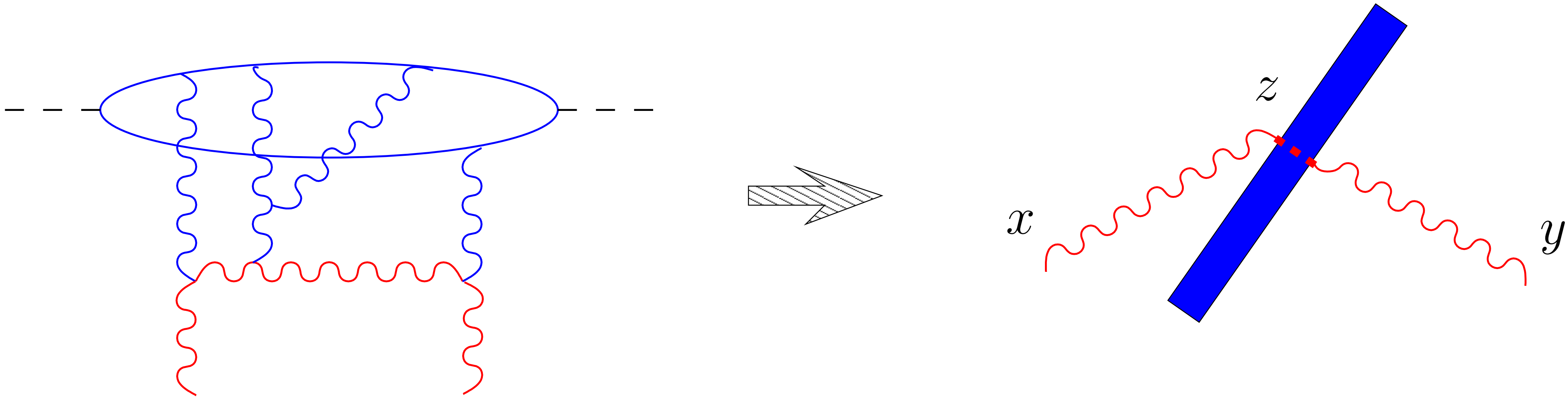
Propagator in the shock-wave background = (free propagator)

$\times$(instantaneous interaction with the shock wave $\sim$ V)$\times$(free propagator)

Covariant vs axial gauge

Covariant gauges: the shock wave is a pancake: $A_+ \sim \delta(x_-)$, $A_- = A_i = 0$.



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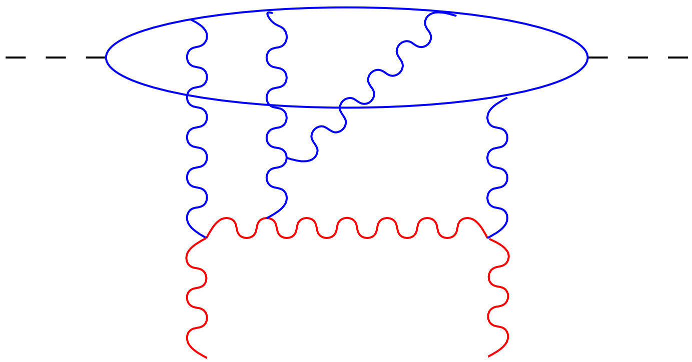
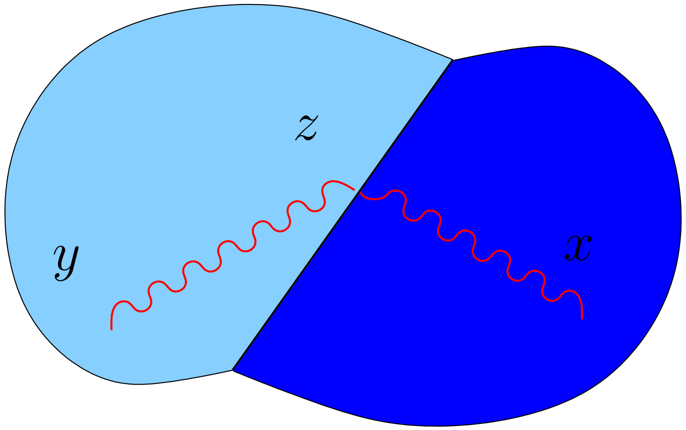
Axial (temporal) gauges: the shock wave is a piece-wise pure gauge

$$A^i = \mathcal{V}_1^i(z_\perp)\theta(z_+) + \mathcal{V}_2^i(z_\perp)\theta(-z_+), \qquad A_+ = A_- = 0, \qquad \mathcal{V}_i \equiv V^\dagger \frac{i}{g}\partial_i V$$





$$G(x,y) = \int dz V_1^\dagger(x_\perp)\frac{1}{(x-z)^2}V_1(z_\perp)\delta(z_-)V_2^\dagger(z_\perp)\frac{1}{(z-y)^2}V_2(y_\perp)$$

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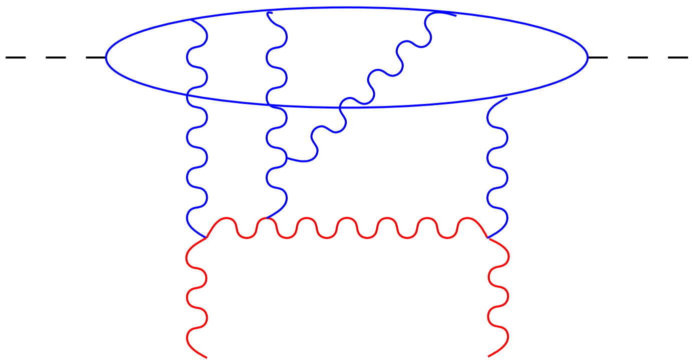

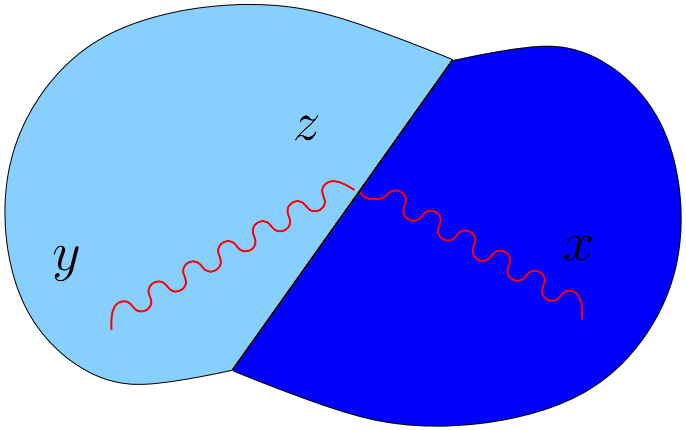

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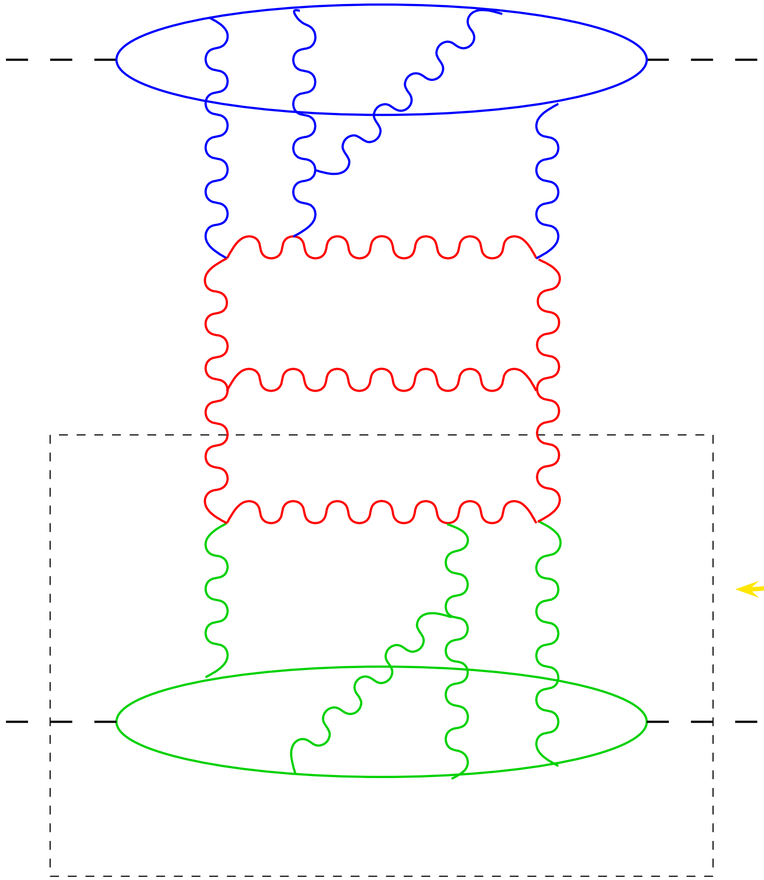
The source for such a field is

$$\exp\{i\int d^2z_\perp\{\mathcal{V}_1^i(z_\perp) - \mathcal{V}_2^i(z_\perp)\}(0, F_{-i}, 0)_z\}$$

$$(0, F_{-i}, 0)_z \equiv \int du[z_\perp, up_1 + z_\perp]F_{-i}(up_1 + z_\perp)[up_1 + z_\perp, z_\perp]$$



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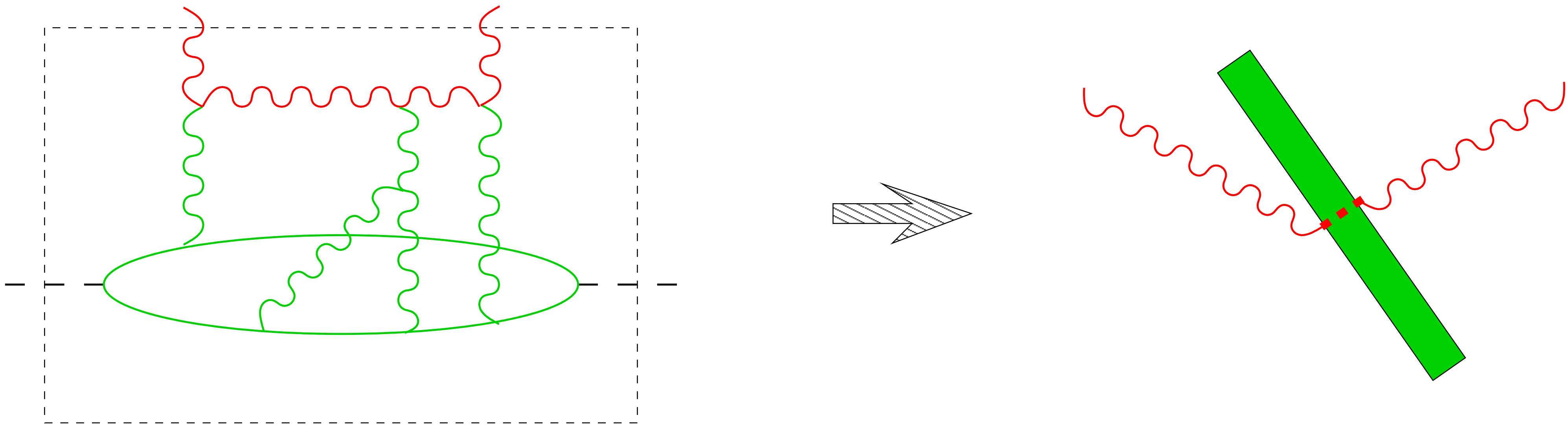
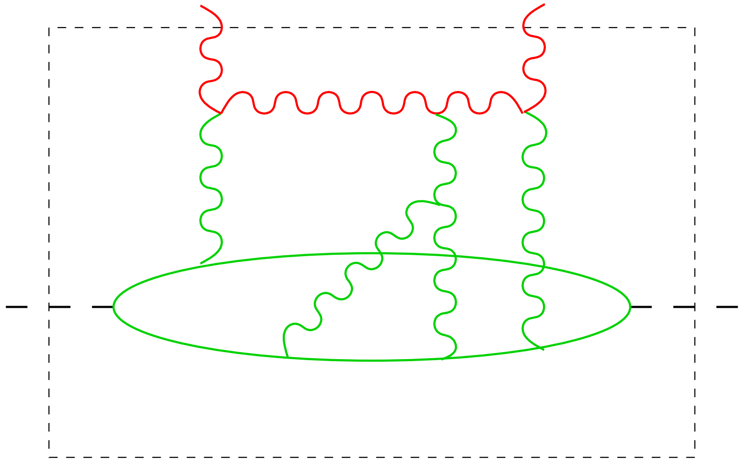
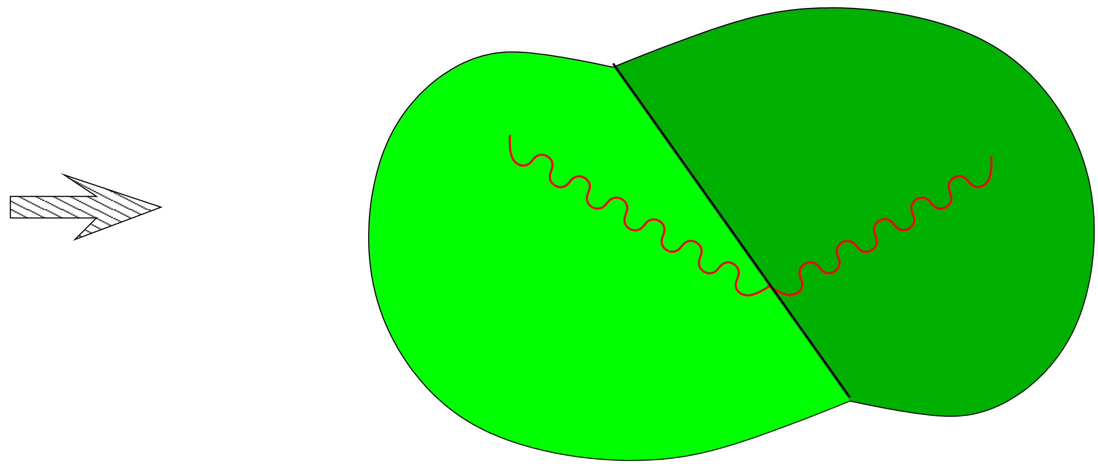


Axial gauges:

$$\mathcal{U}_i \equiv U^\dagger \tfrac{i}{g}\partial_i U$$





The source is

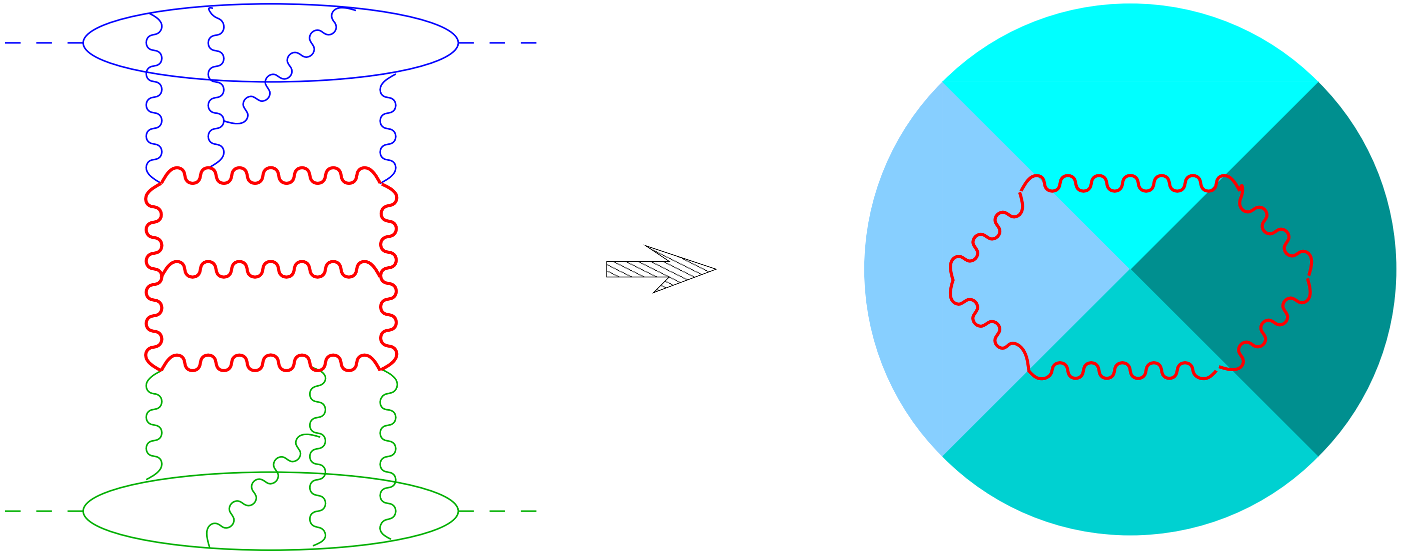
$$\exp\{i\int d^2z_\perp(\mathcal{U}_1^i - \mathcal{U}_2^i)(z_\perp)(0,F_{+i},0)_z\}$$

$$[0,F_{+i},0] \equiv \int du[z_\perp,up_2+z_\perp]F_{+i}(up_2+z_\perp)[up_2+z_\perp,z_\perp]$$
$$= \ [0,\infty p_2]_z i\tfrac{\partial}{\partial z_i}[\infty p_2,0]_z + [0,-\infty p_2]_z i\tfrac{\partial}{\partial z_i}[-\infty p_2,0]_z$$

Scattering of two shock waves

Gluons in the central region of rapidity move in the “external” fields of two shock waves

In the axial gauges



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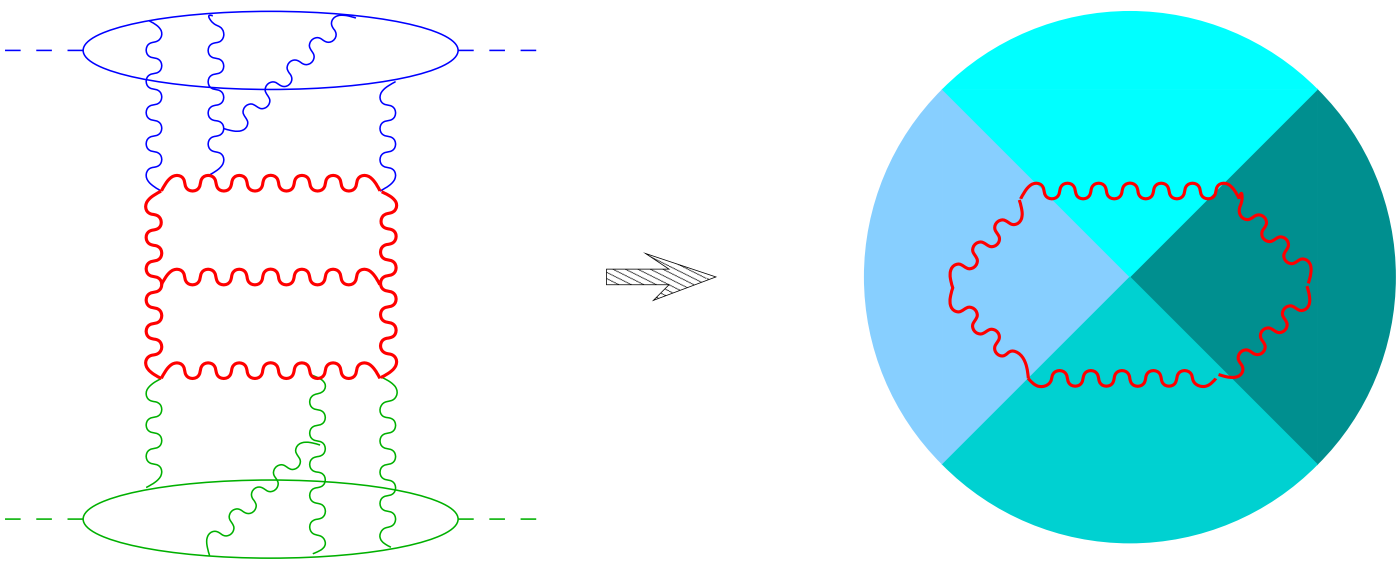
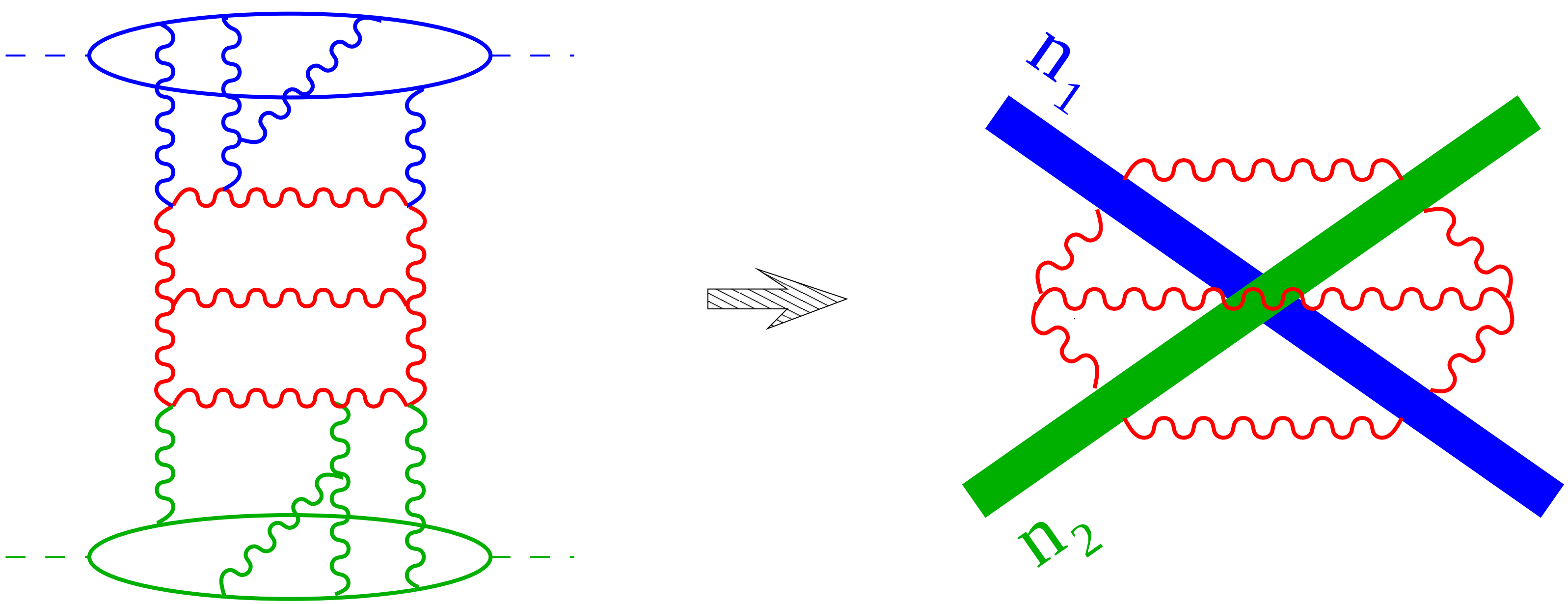


Integration over A fields gives the effective action

$$e^{iS_{\mathrm{eff}}(U_i,V_i,\Delta\eta)}=\int DA\; e^{iS_{\mathrm{QCD}}(A)+i\int d^2z_\perp\{(\mathcal{V}_1^i-\mathcal{V}_2^i)_z(0,F_{-i},0)_z+(\mathcal{U}_1^i-\mathcal{U}_2^i)_z[0,F_{+i},0]_z\}}$$

Rapidity $\Leftrightarrow$ slope of the Wilson line $\Delta\eta = \eta_1 - \eta_2$



$$e^{iS_{\rm eff}(U_i,V_i,\Delta\eta)} = \int DA\ e^{iS_{\rm QCD}(A)+i\int d^2z_\perp\{(\mathcal{V}_1^i-\mathcal{V}_2^i)_z(0,F_{-i},0)_z+(\mathcal{U}_1^i-\mathcal{U}_2^i)_z[0,F_{+i},0]_z\}}$$

$$(0,F_{-i},0)_z = (0,\infty n_1)_z i\partial_i(\infty n_1,0)_z + (0,-\infty n_1)_z i\partial_i(-\infty n_1,0)_z,$$
$$[0,F_{-i},0]_z = [0,\infty n_2]_z i\partial_i[\infty n_2,0]_z + [0,-\infty n_2]_z i\partial_i[-\infty n_2,0]_z$$

$S_{\rm eff}$ gives the small-x evolution of the Wilson-line operators

BASIC IDEA: $\alpha_s = \alpha_s(Q_s) \ll 1 \Rightarrow$ SEMICLASSICS IS RELEVANT

McLerran & Venugopalan

Classical YM equation

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$$D^\mu F_{\mu\nu} = \frac{\partial}{\partial A_\mu}(\text{sources})$$

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Two methods of the solution on the market:

- Numerical simulations. Venugopalan & Krasnitz
- Perturbative expansion in strength of one of the shock waves

McLerran et al, Kovchegov & Mueller

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$\Leftrightarrow$ expansion in powers of commutators $[U, V]$ (calculated up to $[U, V]^2$)

The expansion in commutators

If $[U, V] = 0$

$$\bar{A}_+ = \bar{A}_- = 0, \quad \bar{A}^i = \mathcal{U}_1^i \theta(x_+) + \mathcal{U}_2^i \theta(-x_+) + \mathcal{V}_1^i \theta(x_-) + \mathcal{V}_2^i \theta(-x_-)$$

= piece-wise pure gauge .

QED-like: no interaction $\Rightarrow$ no particle production

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If $[U,V]\neq 0$ one can take this ansatz

$$\bar{A}_+^{(0)} = \bar{A}_-^{(0)} = 0, \quad \bar{A}^{i(0)} = \mathcal{U}_1^i\theta(x_+) + \mathcal{U}_2^i\theta(-x_+) + \mathcal{V}_1^i\theta(x_-) + \mathcal{V}_2^i\theta(-x_-)$$

as a trial configuration for the classical solution and improve it order by order in $[U,V]$ by calculating Feynman diagrams in the background of the trial configuration.

Linear term (source) for the trial configuration

$$T_\mu \;=\; -D_k^{(0)}([\mathcal{U}^1_{\mu\perp}, \mathcal{V}^k_1] - \mu \leftrightarrow k)\theta(z_+)\theta(z_-) + 3 \text{ similar terms}$$

$$\bar{A}_\mu = \mathcal{U}^\perp_{1\mu}\theta(x_+) + \mathcal{U}^\perp_{2\mu}\theta(-x_+) + \mathcal{V}^\perp_{1\mu}\theta(x_-) + \mathcal{V}^\perp_{2\mu}\theta(-x_-) + Q_\mu$$

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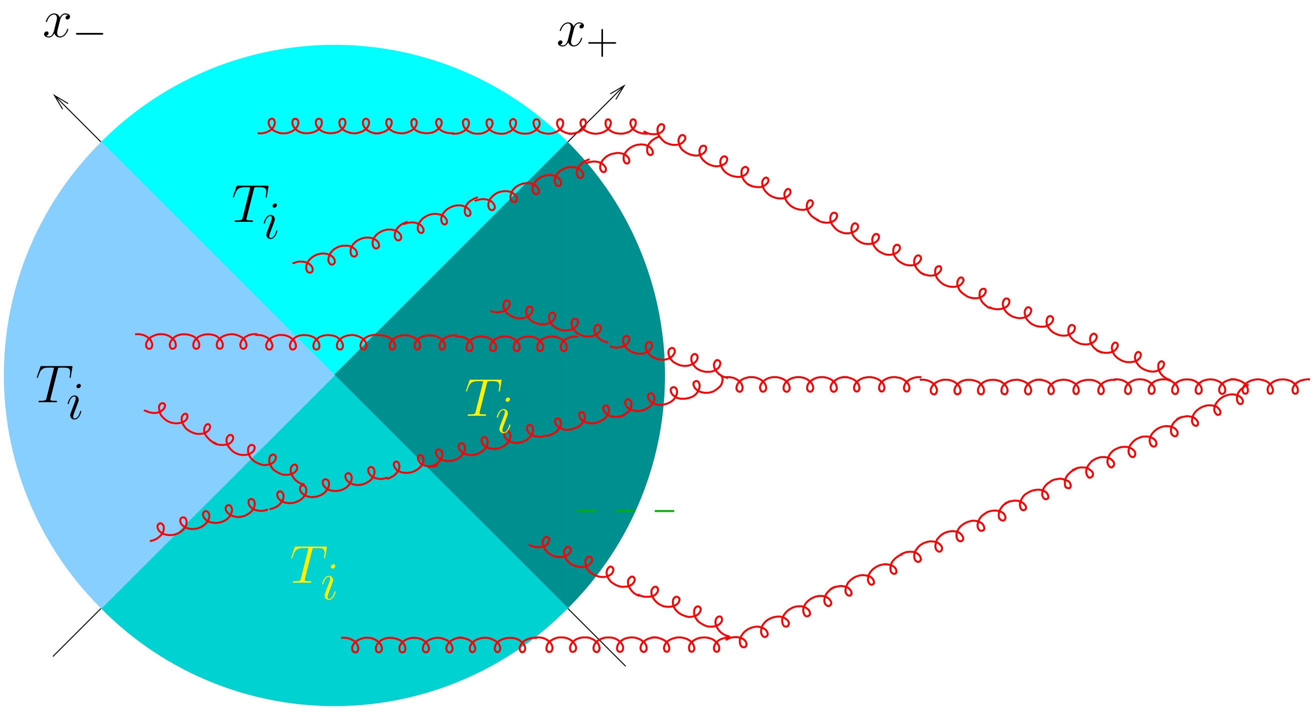
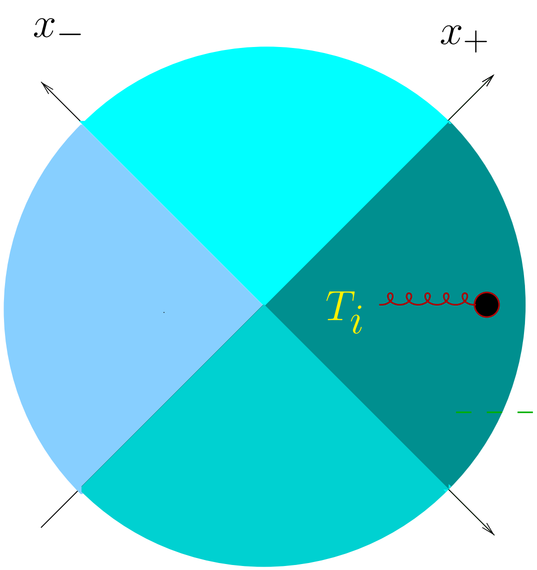
Solve the YM eqn for $Q_\mu(x)$ by iterations $\Leftrightarrow$ calculate Feynman diagrams in the external field $\bar{A}^{(0)}$



1/2-order approximation: a piece-wise pure gauge field



$$
\begin{aligned}
\bar{A}_+ &= \bar{A}_- = 0 \\
\bar{A}^i &= \mathcal{W}^i_F\theta(x_+)\theta(x_-) + \mathcal{W}^i_L\theta(-x_+)\theta(x_-) \\
&+ \mathcal{W}^i_R\theta(x_+)\theta(-x_-) + \mathcal{W}^i_B\theta(-x_+)\theta(-x_-)
\end{aligned}
$$

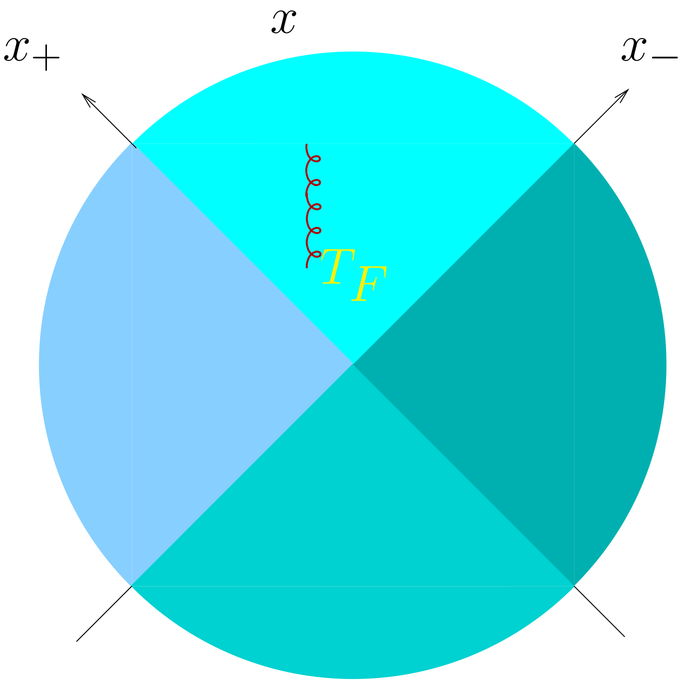
$$
\begin{aligned}
\mathcal{W}^i_F(x_\perp) &= \mathcal{U}^i_1 + \mathcal{V}^i_1 + E^i_F = \text{pure gauge} \\
\mathcal{W}^i_L(x_\perp) &= \mathcal{U}^i_2 + \mathcal{V}^i_1 + E^i_L = \ldots \\
\mathcal{W}^i_R(x_\perp) &= \mathcal{U}^i_1 + \mathcal{V}^i_2 + E^i_R = \ldots \\
\mathcal{W}^i_B(x_\perp) &= \mathcal{U}^i_2 + \mathcal{V}^i_2 + E^i_B = \ldots
\end{aligned}
$$

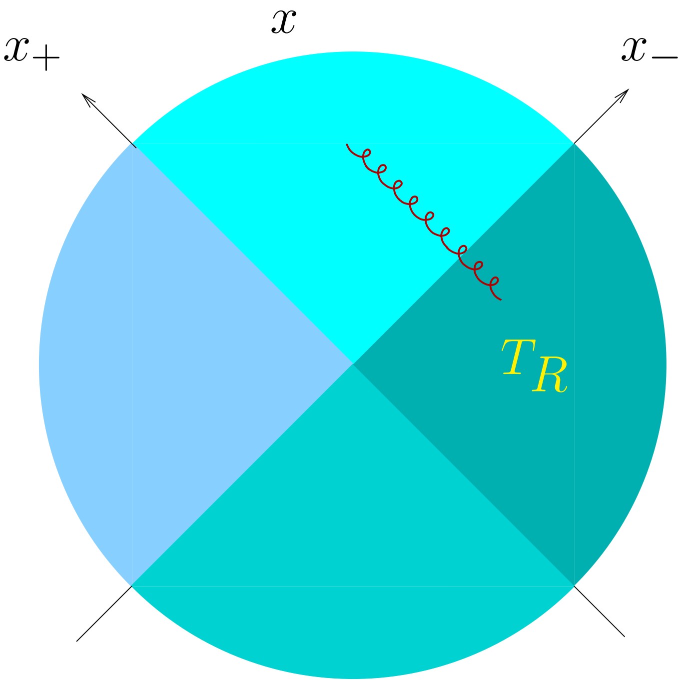
In the first order

$$
E^a_i(x_\perp) = ig\int d^2z(U_xU^\dagger_z + V_xV^\dagger_z - 1)^{ab}\frac{(x-z)^k_\perp}{2\pi^2(x-z)^2_\perp}([\mathcal{U}_i, \mathcal{V}_k]_z - i \leftrightarrow k)^b
$$

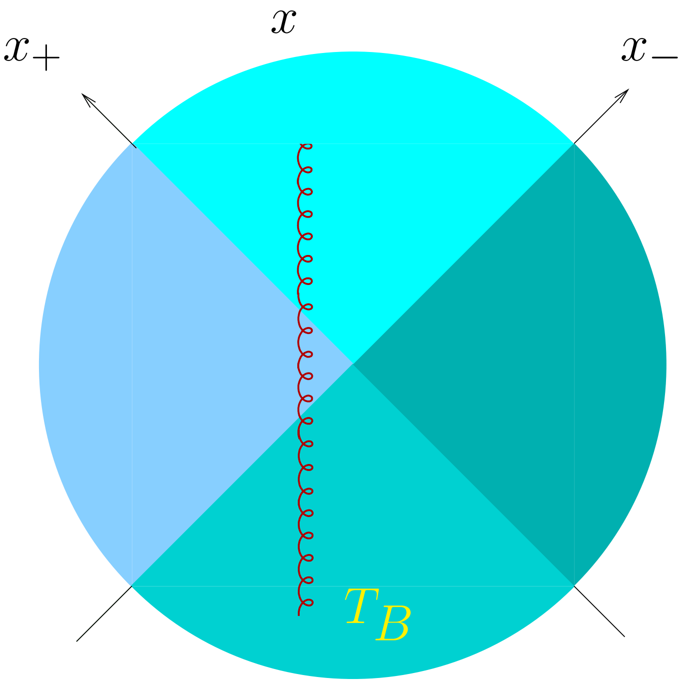
bF gauge $D^\mu Q_\mu = 0 \rightarrow (i\partial_i + g[\mathcal{U}_i + \mathcal{V}_i,)E^i = 0$

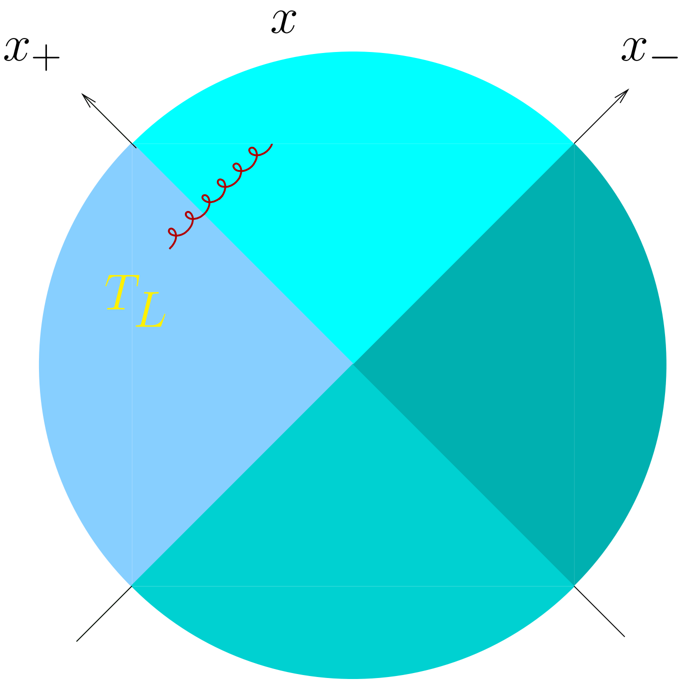
Q_μ in the first order




x
x_+
x_-
T_R

Q_μ in the first order





x_+
x
x_-
T_L

$$Q^{\mathcal{W}_F}_{(1)\mu}(k) \;\equiv\; W_F Q^{\mathcal{W}_F}_{(1)\mu} W_F^\dagger \;=$$
$$\frac{1}{k^2}\Big\{2\frac{p_{1\mu}}{k_-}[\mathcal{V}_{1i} - \mathcal{V}_{2i}, E^i_R - E^i_B] + 2\frac{p_{2\mu}}{k_+}[\mathcal{U}_{1i} - \mathcal{U}_{2i}, E^i_L - E^i_B] + \; 2E^\perp_\mu\Big\}$$

$$E^i \equiv E^i_F - E^i_L - E^i_R + E^i_B = \mathcal{W}^i_F - \mathcal{W}^i_L - \mathcal{W}^i_R + \mathcal{W}^i_B$$

Check: bF gauge condition $(i\partial^\mu + g[\mathcal{W}^\mu_F,)Q_\mu = 0$

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Check: bF gauge condition $(i\partial^\mu + g[\mathcal{W}^\mu_F,)Q_\mu = 0$

Lipatov vertex (effective vertex of gluon emission):

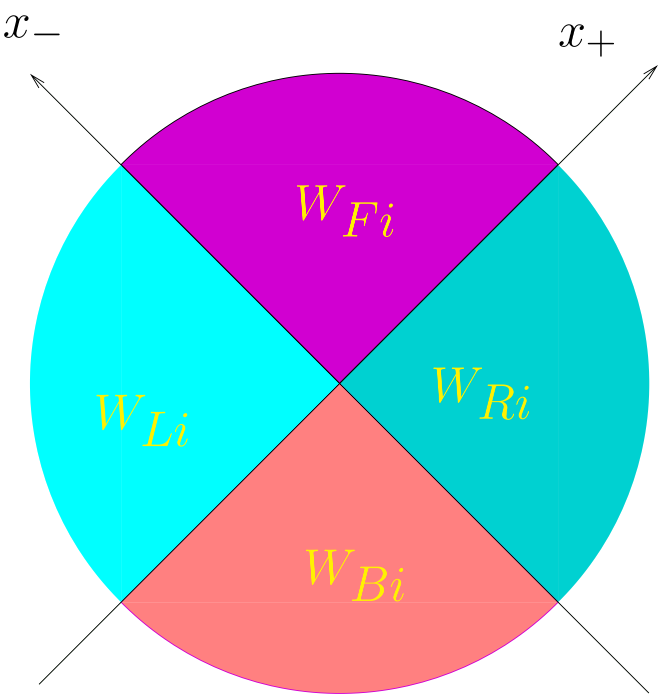
$$L^{(1)}_\mu(k) = k^2 Q^{\mathcal{W}_F}_{(1)\mu}(k)\Big|_{k^2=0} =$$
$$2E^\mu_\perp + 2\frac{p^\mu_1}{k_-}[\mathcal{V}_{1i}-\mathcal{V}_{2i}, E^i_R - E^i_2] + 2\frac{p^\mu_2}{k_+}[\mathcal{U}_{1i}-\mathcal{U}_{2i}, E^i_L - E^i_2]$$

Effective action = product of two Lipatov vertices.

In the $[U,V]^2$ order

$$L^a_\mu L^{a\mu} = 4E^i_a E^{ai}$$

Shortcut to the effective action



The trial configuration:

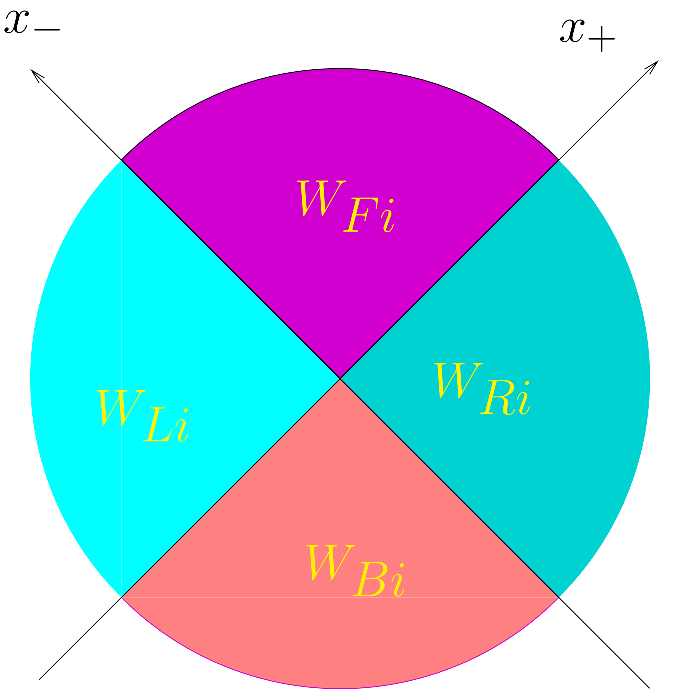
$A_- = A_+ = 0$ and

$$A_i = \theta(x_+)\theta(x_-)\mathcal{W}_{Fi} + \theta(x_+)\theta(-x_+)\mathcal{W}_{R}$$
$$+ \theta(-x_+)\theta(x_-)\mathcal{W}_{Li} + \theta(-x_+)\theta(-x_-)\mathcal{W}_{B}$$

In each of the four quadrants of the space the field is a pure gauge

$$\mathcal{W}_{Fi} = \mathcal{U}_{1i} + \mathcal{V}_{1i} + E_{Fi}$$
$$\mathcal{W}_{Li} = \mathcal{U}_{2i} + \mathcal{V}_{1i} + E_{Li}$$
$$\mathcal{W}_{Ri} = \mathcal{U}_{1i} + \mathcal{V}_{2i} + E_{Ri}$$
$$\mathcal{W}_{Bi} = \mathcal{U}_{2i} + \mathcal{V}_{2i} + E_{Fi}$$

Shortcut to the effective action



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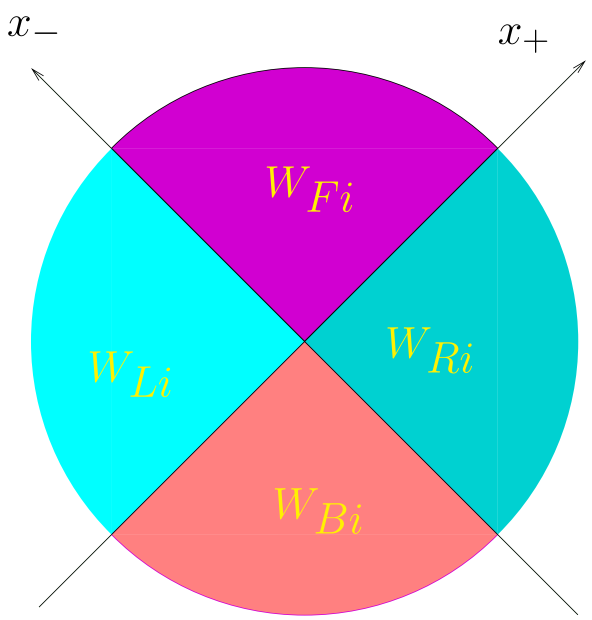
$$\mathcal{W}_{Bi} = \mathcal{U}_{2i} + \mathcal{V}_{2i} + E_{Fi}$$

$$T_i = 2\delta(x_+)\delta(x_-)E_i \quad \Rightarrow$$

$$S_{\text{eff}} = \int dz dz' T_i^a(z) T^{ai}(z') \simeq \alpha_s \Delta\eta \int d^2 z_\perp E_i^a(z_\perp) E^{ai}(z_\perp)$$

Gauge-invariant form of the effective action



$$S_{\text{eff}}(V_1, V_2, U_1, U_2; \Delta\eta) \;=$$

$$(\mathcal{V}_1 - \mathcal{V}_2)^{ai}(\mathcal{U}_1 - \mathcal{U}_2)^a_i \;+\; \frac{\alpha_s \Delta\eta}{4} L^a_i L^{ai}$$

$$\begin{aligned} L^a_i \;&=\; 2(\mathcal{W}_F - \mathcal{W}_L - \mathcal{W}_R + \mathcal{W}_B)^a_i \\ &=\; 2(E_F - E_L - E_R + E_B)^a_i \end{aligned}$$

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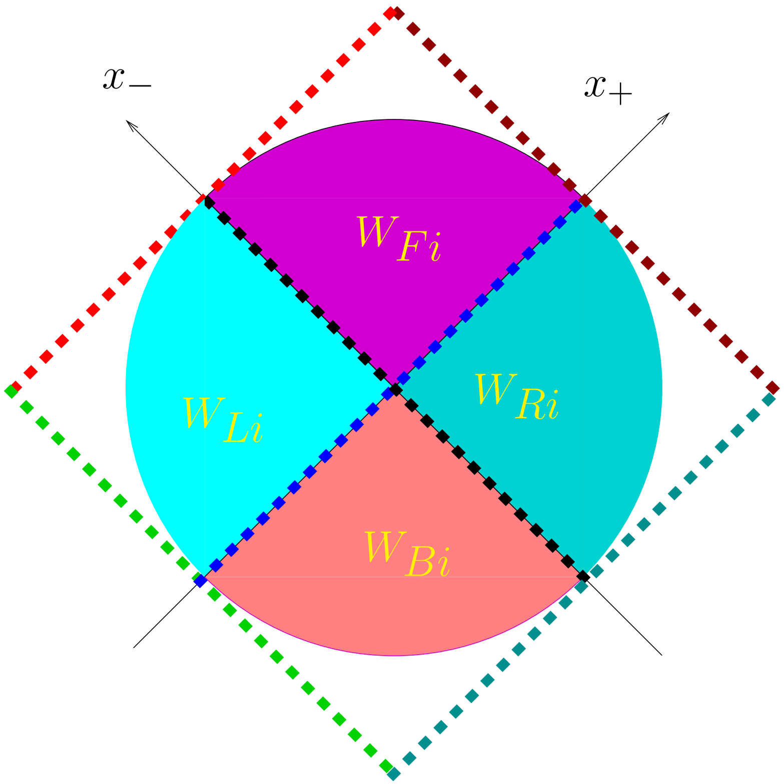

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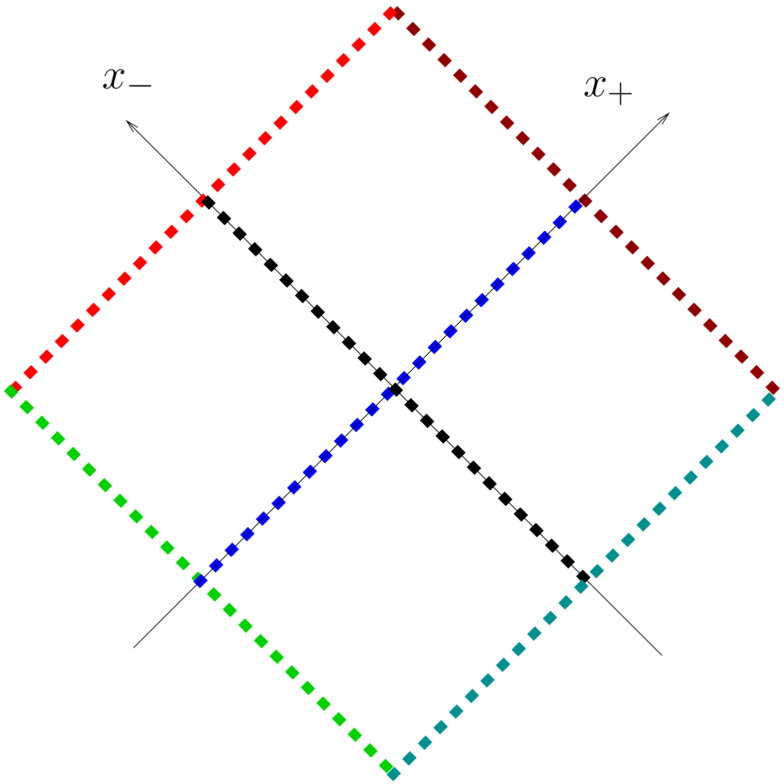
Gauge invariant representation (HIMST):

$$\frac{1}{4} L^{ai} L^a_i = \text{tr}[\infty p_1, F_{-i}, -\infty p_1]_{\infty p_2}[\infty p_2, F_{+i}, -\infty p_2]_{\infty p_1}$$
$$\times [\infty p_1, -\infty p_1]_{-\infty p_2}[-\infty p_2, \infty p_2]_{-\infty p_1} + \text{ cyclic perm.}$$

$$[\infty p_1, F_{-i}, -\infty p_1] \equiv \int_{-\infty}^{\infty} du [\infty p_1, up_1] F_{-i}(up_1)[up_1, -\infty p_1]$$

Gauge-invariant form of the effective action



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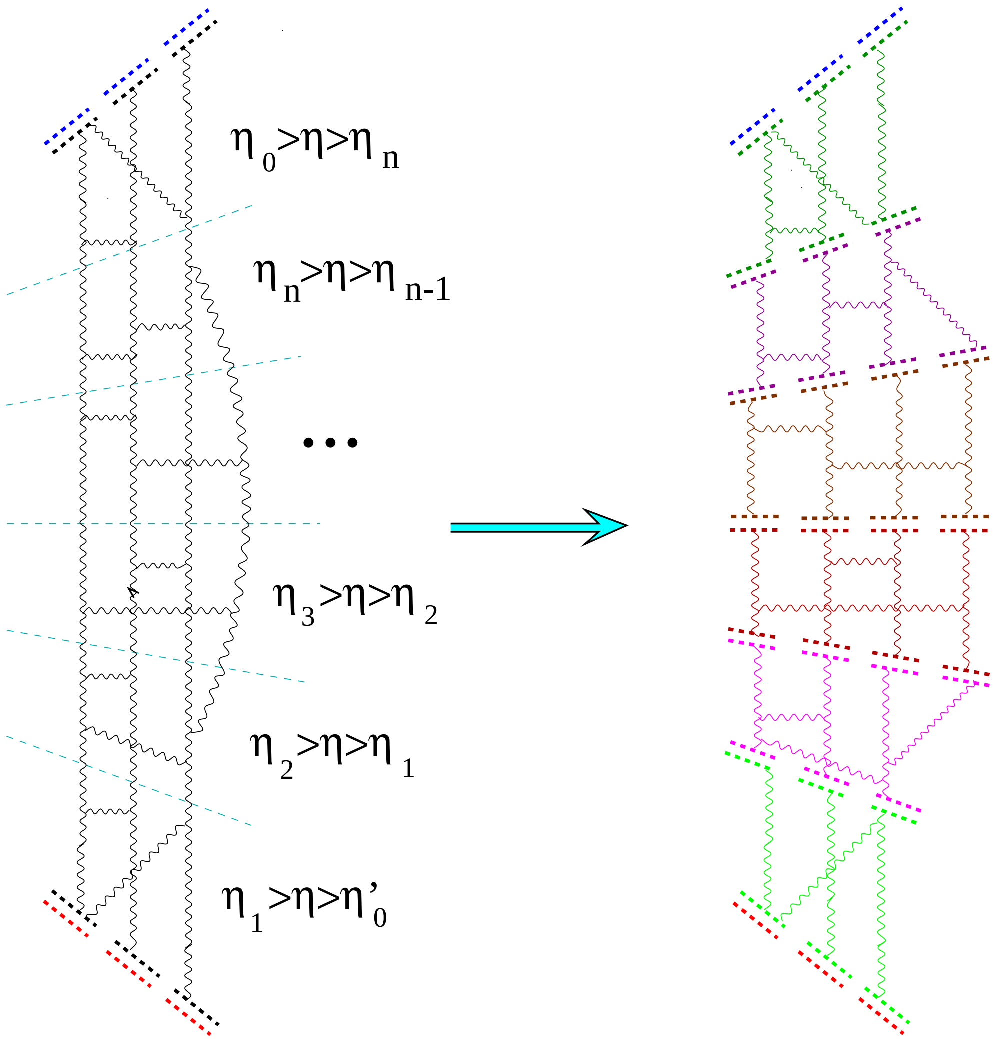
$$\begin{aligned} \frac{1}{4} L^{ai} L^a_i = \mathrm{tr}&[\infty p_1, F_{-i}, -\infty p_1]_{\infty p_2} [\infty p_2, F_{+i}, -\infty p_2]_{\infty p_1} \\ &\times [\infty p_1, -\infty p_1]_{-\infty p_2} [-\infty p_2, \infty p_2]_{-\infty p_1} \;+\; \text{cyclic perm.} \end{aligned}$$

$$[\infty p_1, F_{-i}, -\infty p_1] \equiv \int_{-\infty}^{\infty} du [\infty p_1, u p_1] F_{-i}(u p_1) [u p_1, -\infty p_1]$$

Functional integral over the Wilson-line variables



$$e^{(\mathcal{V}_1^{\eta_n}-\mathcal{V}_2^{\eta_n})_i^a(\mathcal{U}_1^{\eta_n}-\mathcal{U}_2^{\eta_n})^{ai}}$$

$$= \frac{1}{\mathcal{N}}\int D\tilde{U}_1^{\eta_n} D\tilde{U}_2^{\eta_n} D\tilde{V}_1^{\eta_n} D\tilde{V}_2^{\eta_n}$$

$$\exp\Big\{(\mathcal{V}_1^{\eta_n}-\mathcal{V}_2^{\eta_n})_i^a(\tilde{\mathcal{U}}_1^{\eta_n}-\tilde{\mathcal{U}}_2^{\eta_n})$$

$$-(\tilde{\mathcal{U}}_1^{\eta_n}-\tilde{\mathcal{U}}_2^{\eta_n})^{ai}(\tilde{\mathcal{V}}_1^{\eta_n}-\tilde{\mathcal{V}}_2^{\eta_n})^{ai}$$

$$+(\tilde{\mathcal{V}}_1^{\eta_n}-\tilde{\mathcal{V}}_2^{\eta_n})^{ai}(\mathcal{U}_1^{\eta_n}-\mathcal{U}_2^{\eta_n})^{ai}\Big\}$$

$$\int_{\eta_{n-1}}^{\eta_n} DA\, e^{iS+i(\tilde{\mathcal{V}}_1^{\eta_n}-\tilde{\mathcal{V}}_2^{\eta_n})_i^a(\mathcal{U}_1^{\eta_n}-\mathcal{U}_2^{\eta_n})^{ai}+i(\mathcal{V}_1^{\eta_{n-1}}-\mathcal{V}_2^{\eta_{n-1}})_n^a(\tilde{\mathcal{U}}_1^{n-1}-\tilde{\mathcal{U}}_2^{n-1})^{ai}}$$

$$= e^{i(\tilde{\mathcal{V}}_1^{\eta_n}-\tilde{\mathcal{V}}_2^{\eta_n})_i^a(\tilde{\mathcal{U}}_1^{n-1}-\tilde{\mathcal{U}}_2^{n-1})^{ai}+\frac{\alpha_s}{4}L_i(\tilde{\mathcal{V}}^{\eta_n},\tilde{\mathcal{U}}^{n-1})L_i(\tilde{\mathcal{V}}^{\eta_n},\tilde{\mathcal{U}}^{n-1})}$$

Functional integral over the Wilson-line variables

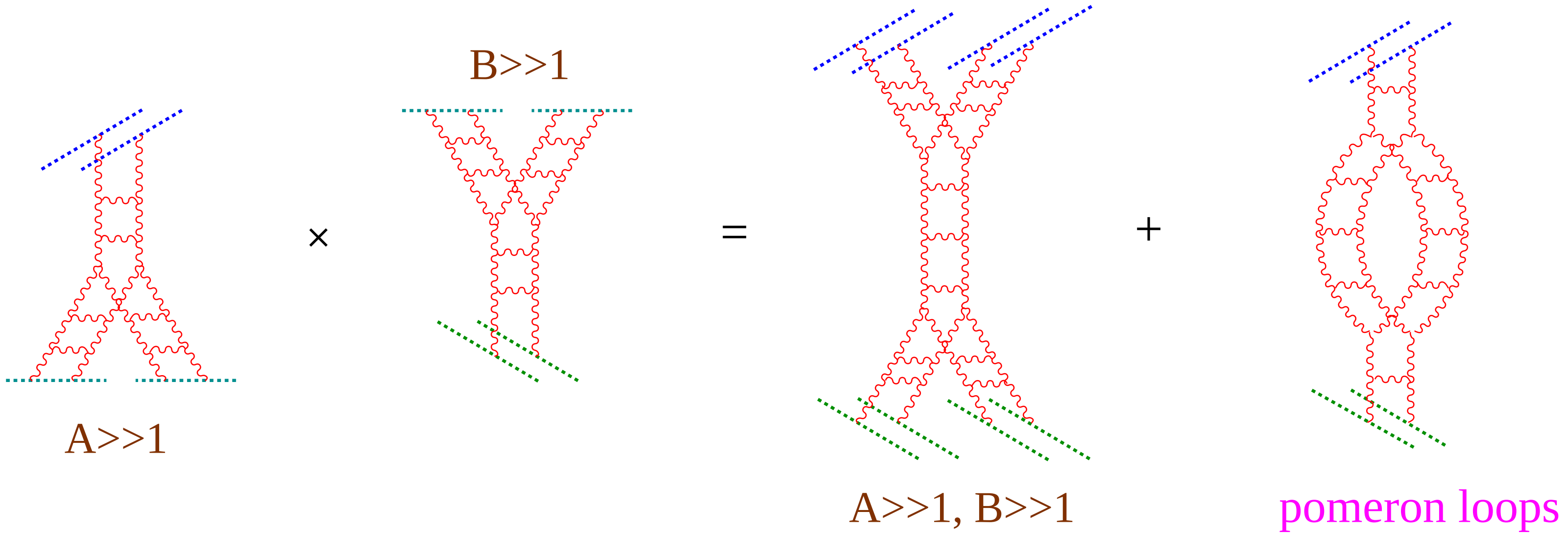
$$e^{iS_{\rm eff}(U_1,U_2,V_1,V_2;\eta_1-\eta_2)} =$$

$$\int_{U_{\eta_2}=U} DU^\eta_{1x} DU^\eta_{2x} DV^\eta_{1x} DV^\eta_{2x}\ e^{i\int d^2x_\perp [(\mathcal{V}_1-\mathcal{V}_2)^a_i(\mathcal{U}_1^{\eta_1}-\mathcal{U}_2^{\eta_1})^{ai}+\int_{\eta_2}^{\eta_1} d\eta \mathcal{L}(U_1,U_2,V_1,V_2,\eta)]}$$

$$\mathcal{L}(U_k,V_k,\eta) = -(\mathcal{V}_1^\eta - \mathcal{V}_2^\eta)^a_i \frac{\partial}{\partial\eta}(\mathcal{U}_1^\eta - \mathcal{U}_2^\eta)^{ai} - i\frac{\alpha_s}{4} L^a_i(U,V) L^{ai}(U,V)]\Big\}$$

L_i is local in terms of W's but unfortunately non-local in terms of U and V.

This formula contains both "upside down" and "bottom up" small-x "fan" evolutions $\Rightarrow$ pomeron loops



Conclusions

- High-energy hadron-hadron scattering $\Leftrightarrow$ collision of two QCD shock waves (Color Glass Condensates?)
- For two nuclei, A and B, the expansion in commutators of Wilson lines is a symmetic expansion in both $\frac{B}{A}$ or $\frac{A}{B}$ parameters.
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Outlook

- The $[U, V]^2$ term in $\mathcal{L}$
- Big Q: What is the field produced by the collision (in all orders in $[U, V]$)?
- $\Leftrightarrow$ Big Q: S_{eff} in (in all orders in $[U, V]$) ?